Transitions and transport for a spatially periodic stochastic system with locally coupled oscillators

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In this paper, with a special model, we investigate the spatially periodic stochastic system with locally coupled oscillators subject to a constant force F. A nonequilibrium second-order phase transition is found when $F=0$. This phase transition is reentrant when the additive noise is weak. With varying the constant force F, a continuous or discontinuous transition between the states with positive and negative mean fields ($\mu>0$ and $\mu<0$) is observed, which is not a phase transition. The mean field or current sometimes exhibits hysteresis as a function of F. With the variation of the force F, when hysteresis of the mean field or current versus F appears, a nonzero probability current with definite direction will occur at the point $F=0$. The correlation between the additive and multiplicative noises has an effect on the transitions and the transport.

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I. INTRODUCTION

Noise-induced nonequilibrium phenomena in nonlinear systems have recently attracted a great deal of attention in a variety of contexts [1]. In general, these phenomena involve a response of the system that is not only produced or enhanced by the presence of the noise, but optimized for certain values of the noise. One example is the phenomenon of stochastic resonance [2], wherein the response of a nonlinear system to a signal is enhanced by the presence of noise, and maximized for certain values of the noise parameters. Another is the "Brownian motor," wherein for Brownian motion in stochastic spatial periodic potentials the spatial asymmetry or noise asymmetry leads to a systematic transport whose magnitude and even direction can be turned by the parameters of the noise [3,4]. A third is the nonequilibrium transition for a system with finitely or infinitely coupled oscillators, which is probably a phase transition (first or second order) [5–10] or is not [10,11]. For these systems, the most exciting factor is that a reentrant second-order phase transition was found for a general spatially extended model by Van den Broeck *et al.* [6]. Afterward, this phenomenon was found in many systems with coupled oscillators. A fourth such phenomenon is resonant activity [12]. Here the mean first passage time (MFPT) of a particle driven by (usually white) noise over a fluctuating potential barrier exhibits a minimum as a function of the parameter of the fluctuating potential barrier (usually the flipping rate of the fluctuating potential barrier).

In this paper, we will study the properties of the spatially periodic stochastic system with locally coupled oscillators subject to a constant force. Reentrant phase transition of the system may occur. The transport driven by the constant force will be analyzed in detail. The problems will be set out as follows. First, we consider a general model of locally coupled oscillators. Then using the formulas derived by us, a special example will be investigated.

II. SPATIALLY PERIODIC STOCHASTIC SYSTEM WITH LOCALLY COUPLED OSCILLATORS

The equations of the overdamped Brownian particles for the system are (in dimensionless form and in the Stratonovich sense)

$$\dot{x}_i = f(x_i) + g(x_i)\xi_i(t) - \frac{D}{2d}\sum_j (x_i - x_j) + \eta_i(t) + F,$$

$$f(x_i) = -\frac{dU_1(x_i)}{dx_i}, \tag{1}$$

$$g(x_i) = -\frac{dU_2(x_i)}{dx_i},$$

in which the variables x_i are defined on lattice points i ($i=1,2,3,\ldots,L^d$) of a cubic in d dimensions. $U_1(x_i)$ and $U_2(x_i)$ are spatially periodic functions of x_i with period $b-a=L$. $\xi_i(t)$ and $\eta_i(t)$ are Gaussian white noises with $\langle\xi_i(t)\rangle=\langle\eta_i(t)\rangle=0$, $\langle\xi_i(t)\xi_j(t')\rangle=2D_1\delta_{ij}\delta(t-t')$, $\langle\eta_i(t)\eta_j(t')\rangle=2D_2\delta_{ij}\delta(t-t')$, and $\langle\xi_i(t)\eta_j(t')\rangle=2\lambda\sqrt{D_1D_2}\delta_{ij}\delta(t-t')$ with $-1\leqslant\lambda\leqslant 1$. D is a coupling constant and F a constant force. The oscillators we are considering are infinite, and we have introduced the Weiss mean-field approximation $\mu\doteq\langle x\rangle=\overline{F}(\mu)$, which has been extensively applied [5,6,9,13]. In this approximation, all the oscillators have an identical evolution given by the nonlinear stochastic equation with the mean field μ, and the approximate equation of Eq. (1) is

$$\dot{x} = f(x) + g(x)\xi(t) - D(x-\mu) + \eta(t) + F. \tag{2}$$

Equation (2) yields the Fokker-Planck equations [9,14],

$$\partial_t P(x,\mu,t) = -\partial_x J(x,\mu,t), \tag{3}$$

with the probability current $J(x,\mu,t)$ given by

$$J(x,\mu,t) = A(x,\mu)P(x,\mu,t) - \partial_x B(x)P(x,\mu,t), \qquad (4)$$

where $A(x,\mu)=f(x)-D(x-\mu)+D_1g(x)g'(x)+\lambda\sqrt{D_1D_2}g'(x)+F$ and $B(x)=D_1g^2(x)+D_2+2\lambda\sqrt{D_1D_2}g(x)$. In the stationary state $t\to\infty$, the distribution $P(x,\mu,t)\to P(x,\mu)$, and the current $J(x,\mu,t)\to J=$const. Then we have

$$J = A(x,\mu)P(x,\mu) - \partial_x B(x)P(x,\mu). \qquad (5)$$

The periodic boundary condition for the system is $P(a,\mu)=P(b,\mu)$. For convenience, we define $\Phi(x,\mu)=\int_a^x A(x',\mu)/B(x')dx'$. It is easy to obtain

$$J = N[1 - e^{\Phi(b,\mu)}], \qquad (6)$$

where $N(\mu)=P(b,\mu)B(b)/\int_a^b e^{-\Phi(x',\mu)}dx'$, which is the normalized constant for the stationary probability distribution. The stationary solution of Eq. (3) is

$$P_{st}(x,\mu) = \frac{N(\mu)e^{\Phi(x,\mu)}}{B(x)}\int_a^b e^{[-\Phi(x',\mu)-\Phi(b,\mu)\theta(x-x')]}dx', \quad (7)$$

where $\theta(x-x')$ is the Heaviside step function [For a more detailed derivation of formulas (3)–(7), see Ref. [4]]. Now the Weiss mean-field approximation is [5,6,9,13]

$$\mu \doteq \langle x\rangle = \overline{F}(\mu) = \int_a^b xP_{st}(x,\mu)dx. \qquad (8)$$

Below we discuss the transition and transport of particles for the system. (i) If $F=0$, in the presence of spatial symmetry, Eq. (8) always has a trivial solution $\mu=0$; with the appearance of multiple solutions, we can find $\mu\neq 0$. If $\lambda\neq 0$, as long as $f(x)$ is odd and $g(x)$ even, it follows from Eq. (1) that any realization $\{x_i(t)\}$ is equally probable as $\{-x_i(t)\}$, so the symmetry-breaking phase transition exists; if $\lambda=0$, when $g(x)$ is odd or even and $f(x)$ odd, the symmetry-breaking phase transition exists. With the zero mean field $\mu=0$, the system must have probability current $J=0$; with nonzero mean field $\mu\neq 0$, J will not be equal to zero. (ii) If $F\neq 0$, the particles will move along the direction of the external constant force F. The system will only have the state $\mu\neq 0$ with asymmetry.

III. A SPECIAL MODEL

In this section, we study an example with $f(x_i)=-\omega_0\sin x_i$ and $g(x_i)=-\sin x_i$ (in dimensionless form) with the period $L=2\pi$. In the stationary state $t\to\infty$, from the corresponding formulas in Sec. II, we can obtain

$$A(x,\mu) = -\omega_0\sin x - D(x-\mu) + D_1\sin x\cos x - \lambda\sqrt{D_1D_2}\cos x + F, \qquad (9)$$

$$B(x) = D_1\sin^2 x + D_2 - 2\lambda\sqrt{D_1D_2}\sin x. \qquad (10)$$

First, we study the case of $F=0$. We find that $f(x)$ and $g(x)$ are odd functions of x. When $\lambda=0$, this corresponds to the condition in which the nonequilibium phase transition happens. It can be easily verified that the function $\overline{F}(\mu)$

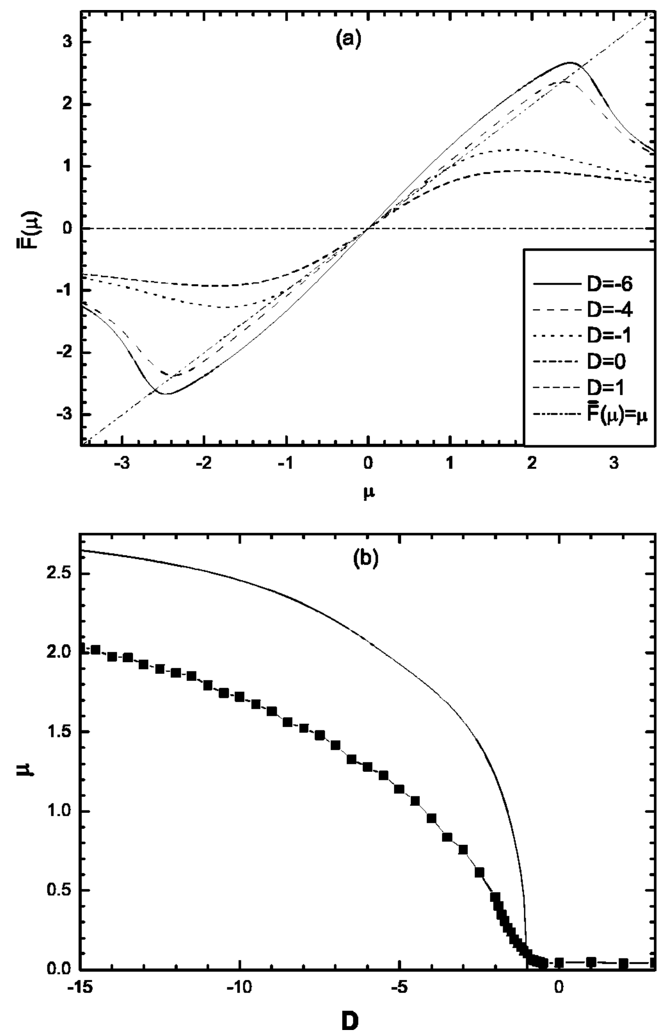

FIG. 1. (a) The function $\overline{F}(\mu)$ vs the mean field μ for different values of $D=-6$, -4, -1, 0, and 1 with $F=0$, $\lambda=0$, $\omega_0=1$, $D_1=1$, and $D_2=0.2$. J, λ, ω_0, D, D_1, and D_2 are dimensionless. The function $\overline{F}(\mu)=\mu$ has one or three solutions. (b) Order parameter μ vs coupling D of the oscillators for $D_1=D_2=0.5$, $\lambda=0$, $\omega_0=0$, according to the mean-field theory (full line), and 2d simulations for system size 64×64. Notice that although the general features of mean-field approximations agree with the simulation result, they tend to overestimate the ordered region.

$=\int_a^b xP(x,\mu)dx$ is a smooth and odd function. Now we turn to a more detailed analysis of the equation $\mu=\overline{F}(\mu)$. When $\partial_\mu\overline{F}(\mu)|_{\mu=0}\leqslant 1$, the function $\overline{F}=\overline{F}(\mu)$ crosses the function $\overline{F}=\mu$ at $\mu=0$ (stable); when $\partial_\mu\overline{F}(\mu)|_{\mu=0}>1$, the function $\overline{F}=\overline{F}(\mu)$ crosses the function $\overline{F}=\mu$ at $\mu=0$ (unstable) and $\mu=\pm\mu^0$ (stable). We plot the function $\overline{F}=\overline{F}(\mu)$ versus μ with $D_1=D_2=1$, $\omega_0=1$, $\lambda=0$, and $D=-6$, -4, -1, 0, 1 in Fig. 1(a). It is clear that the state $\mu\neq 0$ is bistable with $\mu=\pm\mu^0\times(\mu^0>0)$. For the trivial solution $\mu=0$, the system is symmetrical. With the appearance of multiple solutions, we can find an "ordered" phase with an order parameter $\mu=\langle x\rangle\neq 0$. Now the symmetry of the system will be broken. The condition under which the system transits from state $\mu=0$ to state $\mu\neq 0$ or vice versa is $\partial_\mu\overline{F}(\mu)|_{\mu=0}=1$ with $\lambda=0$.

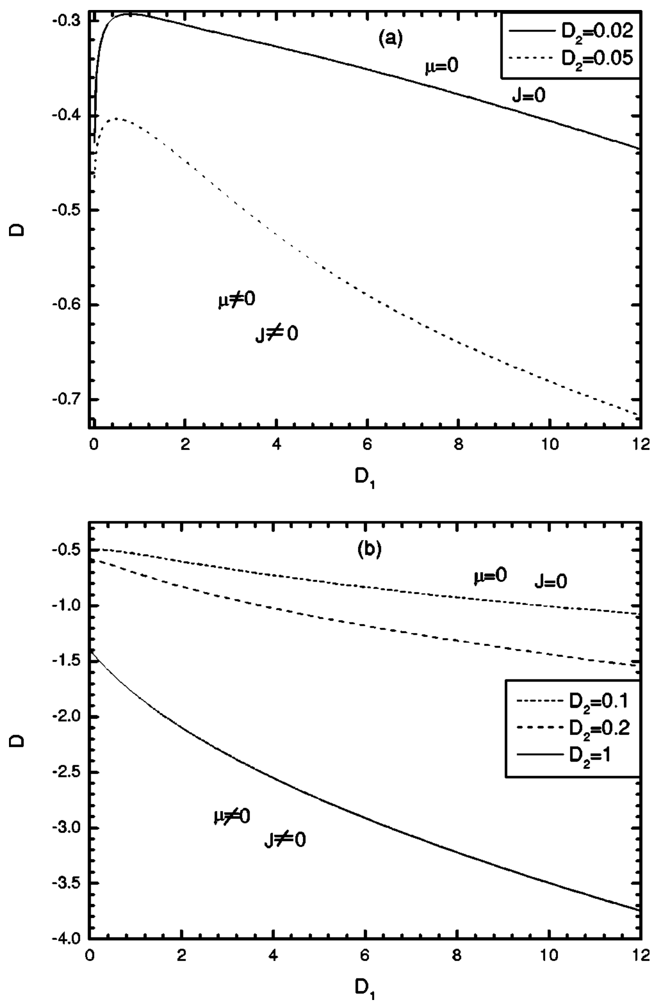

FIG. 2. The phase-transition lines in the case of $F=0$ and $\lambda=0$ for $\omega_0=1$. (a) The coupling constant D vs the multiplicative noise D_1 with $D_2=0.02$ and 0.05. Now the reentrant phase transition occurs with weak additive noise. (b) The coupling constant D vs the multiplicative noise D_1 with $D_2=0.1$, 0.2, and 1. No reentrant phase transition occurs in this case.

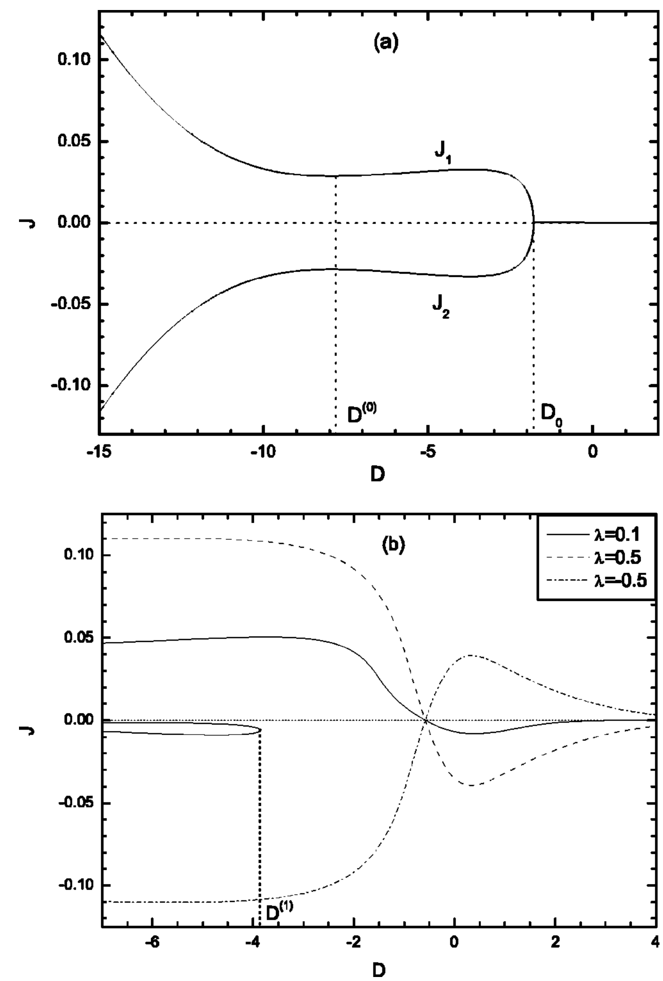

FIG. 3. The probability current J vs the coupling constant D in the case of $F=0$ with $\omega_0=1$, $D_1=D_2=1$. (a) $\lambda=0$. Two currents occur with the same value and opposite direction. (b) $\lambda\neq 0$. When the value of λ is small ($\lambda=0.1$), two asymmetrical stable probability currents occur. When $\lambda=0.5$ or -0.5, only one stable probability current is left with a definite value and direction.

We have performed numerical simulations of the model defined by Eq. (1) on a square lattice. The simulation confirms qualitatively all the results of the mean-field approaches. In Fig. 1(b), we plot the order parameter μ as a function of D for $D_1=D_2=0.5$ according to the mean-field theory developed together with the simulation result. The simulation data indeed confirm the existence of transition.

The transition diagrams for coupling constant D versus the multiplicative noise D_1 are plotted in Figs. 2(a) and 2(b). The transition lines have the following characteristic features. (i) The transition is a second-order phase transition, since there is symmetry breaking and the order parameters change continuously. (ii) The region above the curve corresponds to the state with zero mean field and that below the curve to the state with nonzero mean field. (iii) When the intensity of the additive noise D_2 is lower, such as $D_2=0.02$, 0.05 in Fig. 2(a), the reentrant noise-induced nonequilibrium phase transition appears, in which the appearance of the reentrant of the disordered phase state results from a nontrivial cooperative effect among the additive noise, the multiplicative noise, and the nonlinearity of the system [6]. (iv) When the strength of the additive noise increases, such as $D_2=0.1$, 0.2, and 1 [see Fig. 2(b)], the reentrant of the phase transition disappears. The noises and the coupling of the oscillators have different effects on the system. When the additive noise is weak, the nontrivial cooperation effect among the additive noise, the multiplicative noise, and the nonlinearity of the system increases the asymmetry property of the system with proper multiplicative noise. Now the "ordered" state will be easy to see with weak coupling of the oscillators [see the peaks of Fig. 2(a)]. With larger additive noise, the multiplicative noise increases the freedom of the system and makes the system more "disordered." The coupling of the oscillators has a contrary effect. So with the intensity of the multiplicative noise increasing, the phase transition happens with large coupling of the oscillators [see Fig. 2(b) and the right part of Fig. 2(a)].

When $\lambda=0$, accompanied with symmetry breaking, the mean field is nonzero, and a nonzero flux will appear. The

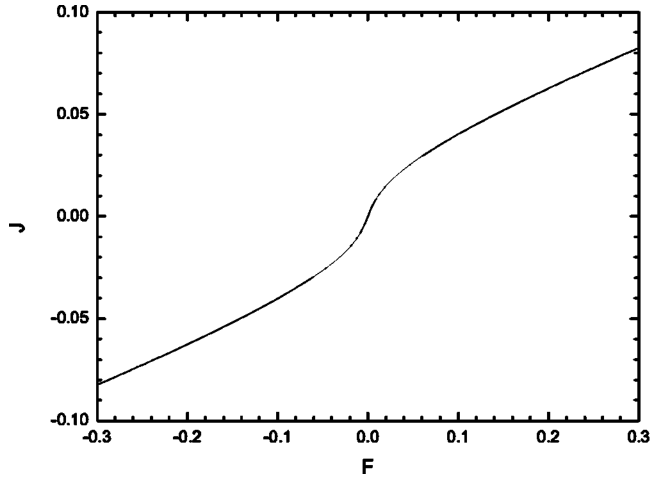

FIG. 4. The probability current J vs the constant force F with weak coupling $D=-1.7$ of the oscillators, for $\omega_0=1$, $\lambda=0$, $D_1=D_2=1$.

probability current J versus the coupling constant D is plotted in Fig. 3(a) with $D_1=D_2=1$ and $\lambda=0$. The figure shows that there is a critical value $D_0=-1.795$ for the appearance of the nonzero probability current. When $D>D_0$, we can get a zero probability current $J=0$. Only on the other side of the point D_0 can we get two symmetrical nonzero probability currents. These currents form a bistable structure, which corresponds to J_1 and J_2 ($J_1=-J_2$) in Fig. 3(a). Only one stable state can occur in the system. It is decided by the initial condition and the evolution of the system which one will occur (J or $-J$). An important property of this figure is that the probability current becomes larger and larger when the value of D becomes smaller than a value $D^{(0)}$ (masked in the figure). This accounts for the strong coupling of the oscillators. The freedom of the system will be decreased with increasing intensity of the coupling among the oscillators. Now the system will likely tend to an "ordered" state. If $\lambda \neq 0$, no nonequilibrium phase transition occurs; only the stable state $\mu \neq 0$ appears. Due to correlation between the additive and multiplicative noises, the symmetry property of the system, and the coupling of the oscillators, the system will be in an "ordered" state. A nonzero mean field and asymmetrical probability current will appear. The probability current J versus the coupling constant D with different nonzero λ ($\lambda=0.1$, 0.5, and -0.5) is plotted in Fig. 3(b). If $\lambda=0.1$, there lie two stable probability currents (the upper and the lower ones of the solid lines) in the region $D<D^{(1)}=-3.85$ (the middle line corresponds to the unstable state of the system); when $D>D^{(1)}$, only one stable probability current is left with a definite value and direction. If $\lambda=0.5$ (or -0.5), there is only one stable nonzero current J. Now the correlation between the additive and the multiplicative noises is strong enough to destroy the bistable structure of the system. From the figure we find that the correlation between the additive and the multiplicative noise is a factor that decreases the freedom of the system. This effect decreases the symmetry property of the system, and an "ordered" state occurs.

If $F\neq 0$, the particles will move along the direction of the force. Such phenomena as hysteresis, negative mobility [15],

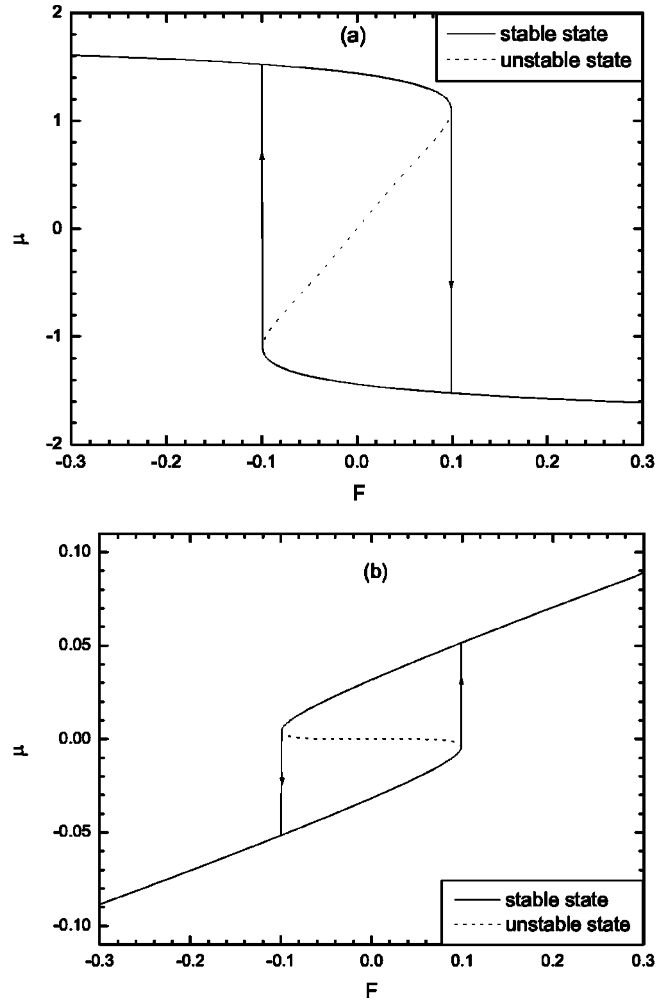

FIG. 5. In the case of large coupling $D=-5$ of the oscillators, with $\omega_0=1$, $\lambda=0$, $D_1=D_2=1$. (a) The mean field μ vs the constant F. Now there is a discontinuous transition line with hysteresis. (b) A discontinuous transport line of J vs F with hysteresis.

and so on, will be expected to appear because of the coupling among different oscillators [16]. First, we consider the case of $\lambda=0$. We have studied the current J as a function of F when the noise strength is definite but the coupling is varied. When the coupling is not large enough, the current J is a continuous function of F (see Fig. 4), which varies continuously from negative to positive or vice versa with the variation of the external force. This current results from a continuous transition of the mean field between the state $\mu>0$ and the state $\mu<0$ as a function of F. If the coupling is large enough, the mean field becomes a discontinuous function of F, such as the line in Fig. 5(a). Now there is a discontinuous transition between the states $\mu>0$ and $\mu<0$, and a hysteresis of the mean field appears [17]. As a result, the probability current J is also a discontinuous function of F [see Fig. 5(b)]. A hysteresis for the current versus the force F appears but no negative mobility. Now with the variation of the external force F, the probability current jumps from a definite direction to the opposite one. Though the appearance of the hysteresis for the current versus the force in Fig. 5(b) results from the hysteresis for the mean field in Fig. 5(a), the direc-

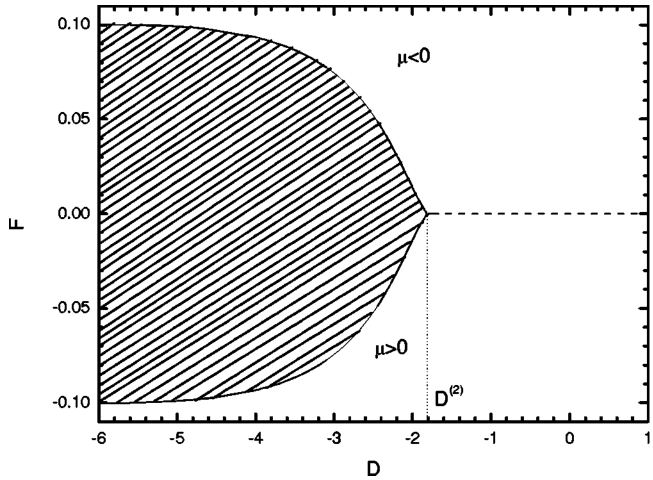

FIG. 6. The transition diagram of the constant force F vs the coupling D of the oscillators for the transition between the state $\mu<0$ and the state $\mu>0$ with $\lambda=0$, $\omega_0=1$, and $D_1=D_2=1$, where the dashed line represents the continuous transition and the solid line represents the discontinuous one. The shadowed region corresponds to the state composed of $\mu>0$ and $\mu<0$.

tions of the hysteresis in these two figures are opposite. In the absence of the constant force F, the two stable mean fields $\pm\mu$ will induce two probability currents $\pm J$. In the presence of the constant force, the nonzero current with a definite direction will occur [see Fig. 5(b)]. For the nonzero current with definite direction when $F=0$, there is a precondition, namely that the constant force changes from nonzero to zero. But this transition is not a first-order phase transition because there is no symmetry breaking of the system. The order parameter μ does not change from zero to nonzero or vice versa. In Ref. [8], although the nonlinear system is analogous to the one in this work, the order parameter $\langle x\rangle$ changes between nonzero and a zero and a first-order phase transition occurs.

The transition line of the constant force F versus the coupling D of the oscillators is given in Fig. 6. The upper region of the figure corresponds to the state $\mu<0$, the lower one to the state $\mu>0$, and the shadowed one to the state composed of $\mu<0$ and $\mu>0$, where hysteresis for the mean field or the nonzero current appears. Some features of this transition can be found from the figure. (i) There is a critical value $D^{(2)}$ of the coupling D. When $D>D^{(2)}$, a continuous transition for the system to transit between the states $\mu>0$ and $\mu<0$ occurs with the variation of the force F. When $D<D^{(2)}$, a discontinuous transition occurs. (ii) The discontinuous transition is doubly unidirectional, which can be observed from Fig. 5(a). (iii) The transition between the two states is symmetric with respect to $F=0$. (iv) The transition is not a phase transition, since with the appearance of the transition there is no symmetry breaking.

Secondly, we consider the case of $\lambda\neq 0$. If correlation between the additive and multiplicative noises is low, such as $\lambda=0.1$, there are still two stable states with large coupling D of the oscillators. Now the diagrams of the mean field μ and the current J versus the external force F are similar to Fig. 5(a) and Fig. 5(b). With nonzero λ, the effect of the term

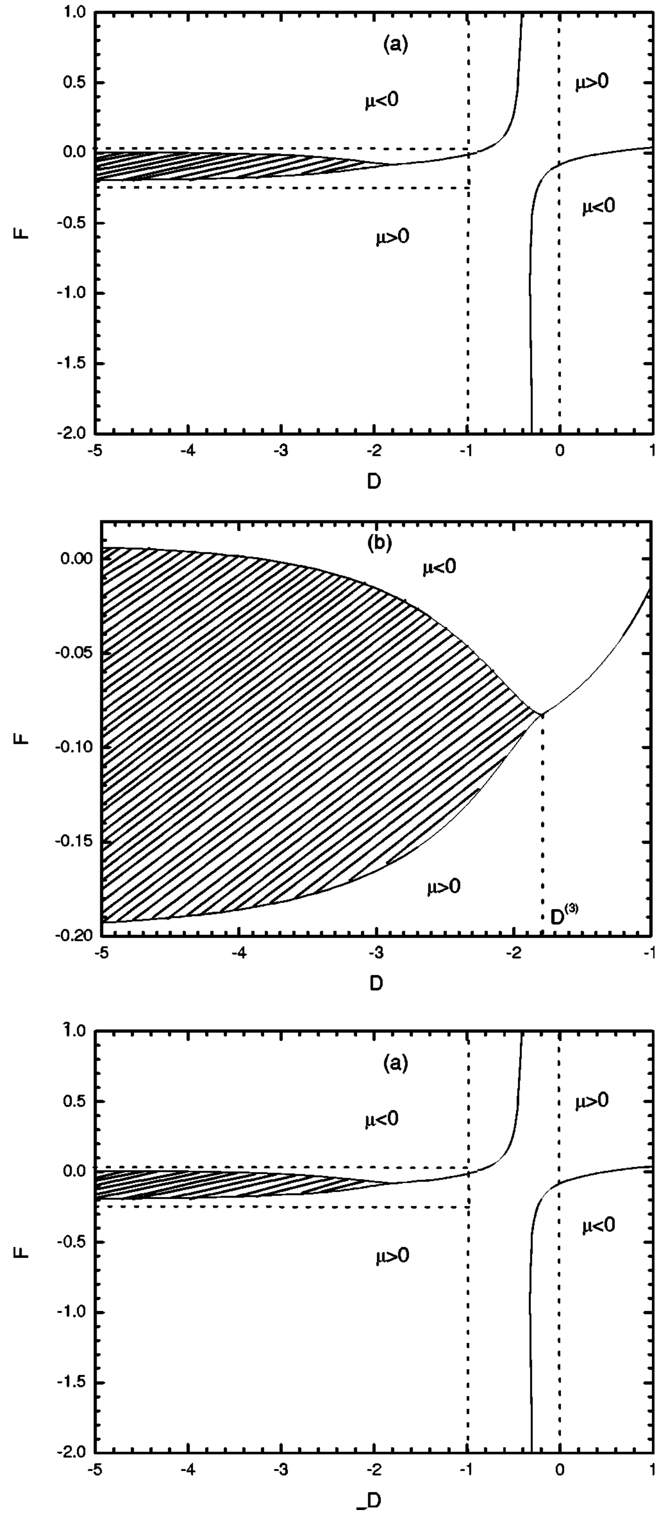

FIG. 7. Transition line of the constant force F vs the coupling constant D with $\lambda=0.1$, $\omega_0=1$, $D=-5$, and $D_1=D_2=1$. (a) Overview of the figure, (b) and (c) enlargements for the corresponding parts (surrounded by dashed lines) in (a).

$-\lambda\sqrt{D_1D_2}\cos x$ in Eq. (9) will be similar to that of the constant force F. The result is that the lines move to the left in the F axis. So the curves will move to the left wholly in comparison with those in Fig. 5(a) and Fig. 5(b). In this case, the transition line becomes more different from that in Fig. 6. In Fig. 7, we plot the transition line for constant force F versus the coupling D of the oscillators with $\lambda=0.1$. Figure 7(a) is an overview of the transition. It is clear that the region in Fig. 7(a) is separated into several parts. In the left region, the intensity of the coupling among the oscillators is high enough for the hysteresis to appear. Figures. 7(b) and 7(c) are enlargements of the corresponding parts in Fig. 7(a). In Fig. 7(b), when $D \leqslant D^{(3)}=-1.79$, it is similar to the corresponding part in Fig. 6. When $D^{(3)} \leqslant D \leqslant -1$, the curve increases abruptly with the increase of D. In Fig. 7(c), this curve increases still, and tends to infinite when $D \to -0.2$. This is the upper line in Fig. 7(c). The lower one has the same property, but it tends to negative infinite with an increase of D around the point $D=-0.2$. Only the middle line varies slowly with an increase of the coupling constant D. These lines separate the figure into several regions. In different regions, the mean field μ has different values. This phenomenon results from the cooperation among the correlation between the additive and multiplicative noises, the coupling of the oscillators, the constant force, and the nonlinearity of the system. If λ is large, such as $\lambda=\pm 0.5$, only one stable state is left. Now the hysteresis for the mean field and the current will not appear.

IV. CONCLUSION AND DISCUSSION

In this paper, we have studied the transition and transport of the spatially periodic system with locally coupled oscillators subject to a constant force F and driven by correlated multiplicative and additive noises. (i) If $F=0$ and $\lambda=0$, there is a phase transition between states $\mu=0$ and $\mu \neq 0$. The transition is a second-order phase transition, since the symmetry of the system has been broken and the order parameter changes continuously. The reentrant phase transition appears when the additive noise is weak, which results from the cooperation among the additive noise, the multiplicative noise, and the nonlinearity of the system. When the strength of the additive noise becomes large, the reentrant of the phase transition disappears. With the nonzero mean field $\mu \neq 0$, we can get the nonzero currents. (ii) If $F=0$ and $\lambda \neq 0$, no phase transition appears and the nonzero current with definite direction will be expected to occur due to the asymmetry produced by the correlation between the additive and multiplicative noises. With large values of λ, the bistable structure of the mean field for the system is destroyed, and there is only one stable probability current. (iii) If $F \neq 0$ and $\lambda=0$, the particles will move along the direction of the force. The hysteresis loop is found with large strength of the coupling among the oscillators. With a variation of the constant force F, when $F=0$, the nonzero current with definite direction occurs. (iv) If $F \neq 0$ and $\lambda \neq 0$, the correlation between the multiplicative and the additive noises cooperates with the constant force F. This cooperation makes the system more complex and increases the nonlinearity of the system. Now, there is still hysteresis for the mean field or current versus the constant force F with the weak correlation between the multiplicative noise and additive noises.

The system with periodic potential is considered here, and the additive noise and the multiplicative noise are correlated with the parameter λ, while in Ref. [18] the nonlinear force $f(x)$ and the function $g(x)$ are not periodic and the additive noise and the multiplicative noise are correlated by a different form. These differences result in a different "effective" potential. In both works, second-order reentrant phase transitions that are introduced by the multiplicative noise have been found. Though there is hysteresis in both works, in our work this phenomenon is caused by the external force that is introduced by us. In the work of Ref. [18], for large enough values of the coupling of oscillators D, a region of coexistence appears in the transition between order and disorder. The additive noise is seen to induce a first-order phase transition in that system. When the first-order phase transition appears, hysteresis can be expected to occur in the coexistence region. The potential considered in our work is periodic. For the mean-field approximation, we consider only from $-\pi$ to π. The system with the function $f(x)=-\sin(x)$ has a more complex nonlinearity. This periodic potential will cause the system to have a lower extent of symmetry, so it will be more difficult to achieve symmetry breaking. A first-order phase transition in the system will be difficult to get.

When there are finite arrays of oscillators, the features of the system will change. For example, in Ref. [19], when the arrays of oscillators are finite, the system has a transition between the state with zero mean field and the state with nonzero mean field, while when the arrays of oscillators are infinite no transition happens in the system. Thus in our paper the case of finite arrays of oscillators remains to be studied.

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